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EVALUATION OF THE LONG TERM STABILITY OF INDUCTORS USING STANDARD ERROR OF ESTIMATE

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Abstract – The aim of this paper is to present a new method used to calculate the long term stability and drift of a group of standard inductors of the Laboratory of Capacitance and Inductance (Lacin), of the National Institute of Metrology, Normalization and Industrial Quality (Inmetro).

Using the concepts of *time series* and *standard error of estimate*, we developed an automated system with the objective of determining the drift (tendency), the long term stability and to forecast future values, what constitutes a powerful tool in the control and in the conservation of measurement standards.

Keywords: stability, time series and standard error of estimate.

I. INTRODUCTION

Usually the standard inductors manufacturer provides the long term stability. This parameter is sometimes the largest contribution to the overall uncertainty of the standard.

In this paper, the stability and drift were estimated by applying the statistical concepts of standard error of estimate and time series. The parameters of interest were calculated based on a database containing the history of periodic calibrations of Lacin standard inductors. The specific statistical techniques used here allow one to know the actual behavior of the standard in the long term and even to make predictions about its future behavior. It was verified, for a group of standard inductors, that the stability estimate is smaller than the value specified by manufacturers. This reduced estimate leads to a smaller overall uncertainty of measurement and consequently to an improvement of calibration processes.

This tool has been very useful in the control and maintenance of Lacin measuring standards.

2. STABILITY AND DRIFT

The stability of a measurement standard may be specified as short-term stability and long-term stability. The ***short-term stability*** is defined as **repeatability** (of results of measurements) - closeness of the agreement between the results of successive measurements of the same measurand carried out under the same measurement conditions. The ***long-term stability*** refers to the ability of a measuring instrument to maintain its metrological characteristics constant along the time. **Drift** is a slow change of a metrological characteristic of a measuring instrument or standard [1].

As previously explained, the statistical calculation of stability and drift are accomplished starting from the data obtained from database of successive calibrations. The data for the calculation of the stability are the date and the measurement result in each calibration. These data correspond to a group of orderly pairs (x_i, y_i), where x_i are the values that correspond to the dates of the experiment and y_i are the calibration results along the time.

3. TIME SERIES

A time series is a collection of observations made sequentially in time or the class of phenomena whose observation process and consequent numeric quantification generates sequences of data distributed in time. Mathematically speaking, one time series is defined by the values Y_1, Y_2, ..., Y_n, of a variable Y (which will be the estimate value for the inductance in our case), at the respective times t_1, t_2, ..., t_n, corresponding to the standard inductors calibrations dates. Therefore, Y is a function of t symbolized by $Y= f(t)$ [2].

3.1. Time Series Forecasting

Time series forecasting is the use of a model to predict future events based on known past events. Or, there is to say, it is to predict future data points before they are measured. The time series show movements or characteristic variations. The analysis of those movements is of great value in several cases; one of them is the forecast of future movements' problem. Thus, if in a time series an independent variable X corresponds to calibration date, the data of interest at different moments will be represented by the values of Y (which are, in our case, the values found in the periodic calibrations of the standard inductors. The straight line or curve of regression of Y on X, in this case it is denominated tendency. It is frequently used with the purpose of estimating or forecasting of future values.

Many time series can be decomposed into the following components: Trend (T): Non-periodic component of time series; Cyclical (C): Periodic component with period longer than seasonal period; Seasonal (S): Recurring pattern (periodic component). Irregular (I): Residual after removing all three components above.
Those movements or components refer to the general direction, according to which seems which the graph of the time series is growing in a long time interval. In general, the methods of time series forecasting, classified as quantitative methods, take as a basis of their forecasts the extrapolation of characteristics of past observations and in the inter-relationship among those observations, supplying accurate forecasts if the future present similar behavior to the past.

4. LINEAR REGRESSION AND STANDARD ERROR OF ESTIMATE

The simple linear regression equation is also called the *least squares* regression equation. Its name tells us the criterion used to select the best fitting line, namely that the sum of the *squares* of the residuals should be *least*. That is, the least squares regression equation is the line for which the sum of quadratic residuals is a minimum.
If there is linear regression we may write the formula for the mean *Y* when *X* is given as

$$(Y = mX + b) \tag{1}$$

Some algebra shows the sum of squared residuals will be minimized by the line for which

$$b = \frac{1}{n}\sum_{j=1}^{n} y_j - \frac{1}{n}\, m \sum_{j=1}^{n} x_j \tag{2}$$

$$m = \frac{n\sum_{j=1}^{n} x_j\, y_j - \sum_{j=1}^{n} x_j \sum_{j=1}^{n} y_j}{n\sum_{j=1}^{n} x_j^2 - (\sum_{j=1}^{n} x_j)^2} \tag{3}$$

The equation (3) can also be written as:

$$m = \frac{n\sum_{j=1}^{n} x_j\, y_j - \sum_{j=1}^{n} x_j \sum_{j=1}^{n} y_j}{n(n-1)\, S_x^2} = \frac{S_{xy}}{S_x^2} \tag{4}$$

Let us consider Y_{est} as the estimated value of *Y* for a given value of *X*. This estimated value can be obtained from

Where is $m = \frac{S_{xy}}{S_x^2}$ called slope and *b* intercept [3,5].

the regression curve of *Y* on *X*, estimated from equation (1), where N is the number of measurements. A measure of the dispersion in relation to the straight line of regression of *Y* on *X* will be given by the formula[4]:

$$\boldsymbol{S_{y.x}} = \sqrt{\frac{\sum_{i=1}^{n}(\boldsymbol{Y_i} - \boldsymbol{Yest_i})^2}{\boldsymbol{N}}} \tag{5}$$

The equation (5) is called the **Standard Error of Estimate** of ***Y*** on ***X***. It is important to note that this Standard Error Estimate has properties analogous to those of standard deviation.

So, we can interpret that this quantity has properties similar to the positive root of the "variance of the noise". Consequently, we can use this equation (5) to evaluate the long term stability of measurement standard. This is especially true in the cases where the graph based on the result of the periodic calibrations presents characteristics similar to the so called “cyclical movement of the time series”.

5. THE SOFTWARE

The software presentation is composed of 4 panels with several fields for identification, data storage and statistical calculations. The first panel is called “history of the calibrations”, where the date of the calibrations, temperatures, conventional true values, uncertainties, serial numbers and other data are stored. The second panel, fig. 1, shows the drift (tendency) quantification of the standard, as well as the estimation of the value of other important statistical parameters. In the lower side of this panel the results estimated for the future calibrations are presented. These estimates are quite useful in the measurements and standards control. Figure 2 shows the third panel of the software, where appears the stability graph, with the values obtained in the periodic calibrations in function of the measurements dates. The example shown is related a 10 H inductor. In the upper left side of the panel there are some adjustments fields. It lower left side shown some results obtained in the statistical calculations. In the lower side of the panel the estimate value for the stability of standard is shown. The last panel (no shown in this paper) is used in the correction of the values for different calibration temperatures.

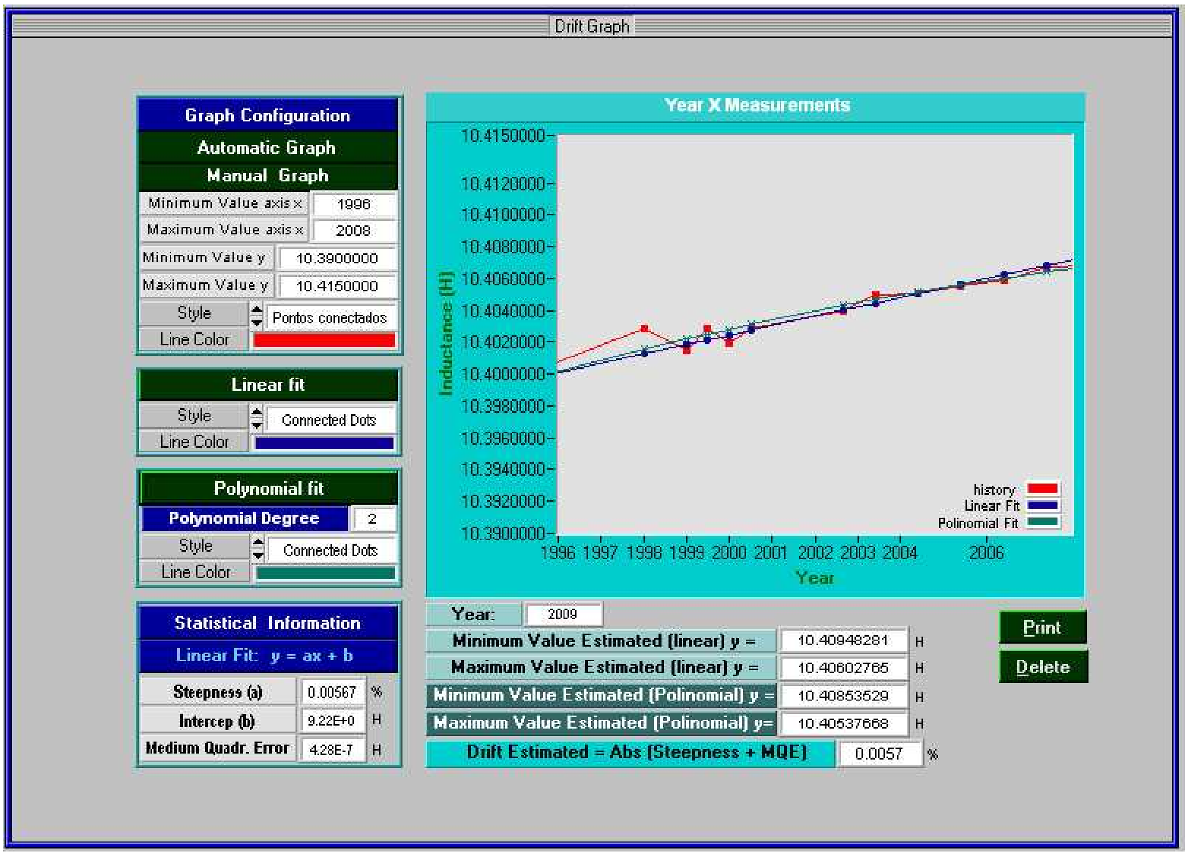

Fig. 1. Panel with the Drift graph estimated values

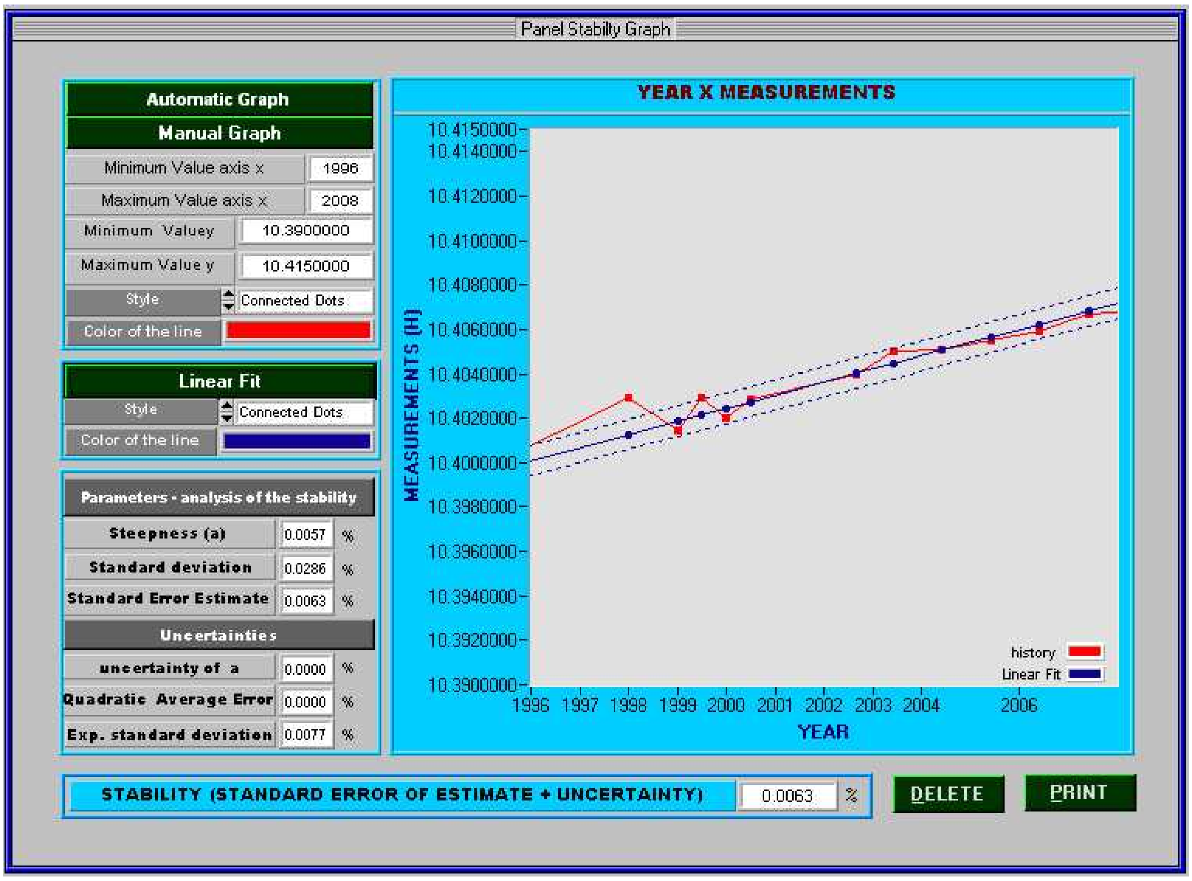

Fig. 2. Panel with the stability graph and other statistical calculations

6. STANDARD INDUCTORS SPECIFICATION

The inductors were chosen based on their nominal values, in order to have an inductor of each value. Besides, only inductors with, at least, 8 regular calibrations where used in the study.

For simplicity, the identification of the inductors was made through the codes used at the own laboratory. Based on this identification, table 1 shows the standards used in this evaluation and some of their main characteristics.

Table 1. Main characteristic of the standard inductors evaluated

Inductor code	Model	Nominal value	Maximum current	Manufacturer stability
A9	1482-B	100 µH	6010 mA	0.01 %
B4	1482-E	1 mH	1890 mA	0.01 %
A5	1482-H	10 mH	600 mA	0.01 %
A6	1482-L	100 mH	192 mA	0.01 %
B7	1482-P	1 H	70 mA	0.01 %
A8	1482-T	10 H	22 mA	0.01 %

7. RESULTS

Table 2 shows the results obtained in the calculation of the stability of Lacin inductance standards. In the second column the values presented are the values declared by the standard inductors manufacturer. The third column shows the calculated values for the slope. The fourth column shows the standard deviation of measures, used by some laboratories to evaluate stability. Finally, the last column shows the evaluation of the stability based on the standard error of the estimate.

Table 2 – Estimate values for the standards under evaluation

Inductor code	Manufacturer stability	Slope	Standard deviation	Estimate stability
A9	0.01 %	0.009 %	0.02 %	0.013 %
B4	0.01 %	0.002 %	0.006 %	0.0043 %
A5	0.01 %	0.002 %	0.01 %	0.0080 %
A6	0.01 %	0.001 %	0.006 %	0.0033 %
B7	0.01 %	0.001 %	0.02 %	0.0018 %
A8	0.01 %	0.006 %	0.01 %	0.0063 %

8. CONCLUSION

The results obtained show that the new method and the automated system developed for the evaluation of the stability of measuring standards are very useful. The calculations show that, except for the inductor A9, in all the others standard inductors the stability estimate is smaller than the value specified by the manufacturer. This reduced estimation leads to a smaller overall uncertainty of measurement and consequently to an improvement of the calibration process.

REFERENCES

[1] "*International Vocabulary of Basic and General Terms in Metrology*, Geneva, International Organization for Standardization, 3rd Edition, 2007.

[2] Spiegel, M. R., "*Statistic*", third edition, pp. 362, MacGraw-Hill, Série Schaum, São Paulo, Brazil 1993.

[3] "*Generation of uncertainties of reducible functions to the first degree adjusted by the method of the least-square*", Brazilian Magazine of Physics vol 21, in the 3, September 1990.

[4] W. J. Dixon and F. J. Massey Jr. "*Introduction to statistical Analysis*", third edition, McGraw-Hill, Kogakusha. Tokio-Japan, 1969.

[5] Cohen, E. R., Tuninsky, U. S., "*The Extended-least-squares Treatment of Correlated Data*", IEEE transactions on instrumentation and measurement", vol 44, no 2, April 1995.