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HIGH-ACCURACY ELECTRICAL MEASUREMENTS USING FRACTIONAL DELAY AND PCA

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Abstract − When applying digital sampling to high-accuracy electrical measurements, aliasing and quantization errors contribute significantly to increase the incertitude. Fractional delay is a simple technique to reduce both errors, improving the accuracy of electrical digital measurements. In this paper we apply the fractional delay technique to asynchronous data acquisition and harmonic estimation of periodic signals. We also combine the use of the fractional delay technique with Principal Component Analysis. We present simulations and laboratory measurements to illustrate both techniques.

Keywords: Electrical Metrology, Fractional Delay, Principal Component Analysis, Sampling Systems

1. INTRODUCTION

Digital sampling is often used in electrical measurements. Although it is a fast and relatively simple method, for high-accuracy measurements errors in digital sampling can be significant [1],[2]. Methods to estimate and to reduce these errors are a main concern in electrical metrology.

When analog-to-digital converters (ADC) are applied to low-frequency AC voltage measurements, two important and correlated error sources are aliasing and quantization. If we consider, as an example, the high-accuracy measurement of harmonics in low-frequency periodic signals, it is important to reduce quantization error by increasing the resolution of the ADC. At the same time, sampling rate should be high enough to mitigate aliasing distortion. Furthermore, distortion introduced by anti-aliasing filters should be minimized.

The use of a fractional-delay (FD) sampling algorithm allows one to reduce significantly the aliasing error, at the same time avoiding the need of high-sampling rates and distortions in the original signal. The use of fractional delay in sampling systems is not new, having been applied, e.g., to filter design [3].

In this paper we describe the properties of fractional-delay sampling of periodic signals applied to high-accuracy electrical measurements. This paper also describes a technique that combine fractional-delay sampling with Principal Component Analysis (FD-PCA).

In the following sections we describe both techniques; we also present simulation results as well as results obtained using an asynchronous data-acquisition system with a single high-accuracy multimeter operating as ADC, in order to verify accuracy improvements obtained by the proposed techniques.

2. ALIASING ANALYSIS AND FRACTIONAL DELAY

The sampling theorem [1] states that if signal function $f(t)$ is bandlimited with finite energy, it can be completely reconstructed after being sampled, as

$$f(t)=\sum_{k=-\infty}^{\infty} f\left(\frac{k\pi}{\Omega}\right) sinc\left(\frac{\Omega t}{\pi}-k\right) \quad (1)$$

where Ω is the bandwidth and $sinc(x) = sin(x)/x$.

It is a mathematical idealization to assume that a signal function is bandlimited with finite energy and infinite duration. Finite-duration signals are indeed not bandlimited, as stated by the uncertainty principle. The reconstructed signal function $f(t)$ then takes the form

$$f(t)=\sum_{k=-\infty}^{\infty} f\left(\frac{k\pi}{\Omega}\right) sinc\left(\frac{\Omega t}{\pi}-k\right)+R_{\Omega f(t)} \quad (2)$$

where $R_\Omega f(t)$ is the aliasing error for a bandwidth of $\Omega = 2\pi M f_0$, f_0 is the signal frequency, and M is the number of harmonics.

In practice, the signal to be sampled by an ADC is of limited time duration and often possesses a much wider frequency band than that of the converter.

In the following subsection we present a description of a fractional delay sampling technique using discrete Fourier transform (DFT), showing that it is possible to eliminate most of the aliasing while both avoiding an increase of the quantization error and reducing the need of anti-aliasing filters.

2.1. Fractional Delay

Let $f(t)$ be a periodic signal with frequency $\omega_0 = 2\pi f_0$, which can be written as an infinite series of sines and cosines.

$$f(t) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty}\{a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)\} \quad (3)$$

$$F(\upsilon) = \frac{1}{2}a_0\,\delta(\upsilon) + \sum_{k=1}^{\infty}\{a_k + jb_k\}\delta(\upsilon - k\omega_0) \quad (4)$$

where $F(\upsilon)$ is the Fourier transform of $f(t)$ and $a_k, b_k \in \Re$ for all k. Consider that $F(\upsilon)\to 0$ as $\upsilon\to\infty$, strictly decreasing for all $F(\upsilon - k\omega_0) \notin 0$, $\upsilon \notin 0$.

Consider now that $f(t)$ is sampled at $t = nt_s + d_{\ell,m}$, where the rate $t_s = 1/(Nf_0)$, and the delay $d_{\ell,m} = (t_s/2^m)\,\ell$ for $\ell m \in \mathbf{N}$. If $(\ell/2^m) \in \mathbf{N}$, then $d_{\ell,m}$ is an integer delay, else $d_{\ell,m}$ is fractional.

To simplify our analysis, our signal of interest $f(nt_s)$ will consist of a cosine series, with fractional delay $d_{\ell=1,m=1} = t_s/2$, as

$$f_A(nt_s) = \sum_{k=0}^{\infty} a_k \cos\left(k\omega_0 nt_s + k\frac{\pi}{N}\right) \quad (5)$$

for $k\omega_0 d_{l=1,m=1} = k\,\pi/N,\ 0 \le n \le N$.

$$Y_A(\upsilon) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} a_k\,\delta(\upsilon - k\omega_0)\,e^{j\frac{k\pi}{N}} + R_\Omega(\upsilon) \quad (6)$$

where $Y_A(\upsilon)$ is the discrete Fourier transform of $f_A(nt_s)$ and $R_\Omega(\upsilon)$ denotes the spectrum extension beyond region $[-N/2\ \omega_0,\ N/2\ \omega_0]$, that will cause aliasing error in the harmonic estimation. As described in [1], the frequency spectrum $Y_A(\upsilon)$ of $f_A(nt_s)$ will present alias, that is, due to under-sampling, reflections of the signal spectrum will overlap.

Considering only the right side of the spectrum, the harmonics with frequencies in the interval $[(N/2+1)\omega_0,\ N\omega_0]$ are reflected in the region $[0,\ N/2\ \omega_0]$ with a frequency translation of 2π from the original signal as

$$Y_1(\upsilon) = \sum_{k=1}^{\frac{N}{2}-1} a_{N-k}\,\delta(\upsilon - k\omega_0)\,e^{j\pi(1+\frac{k}{N})} \quad (7)$$

Whereas the harmonics with frequencies in the interval $[(3N/2-1)\omega_0,\ N\omega_0]$ are reflected with a frequency translation of 2π as

$$Y_2(\upsilon) = \sum_{k=1}^{\frac{N}{2}-1} a_{N+k}\,\delta(\upsilon - k\omega_0)\,e^{j\pi(-1+\frac{k}{N})} \quad (8)$$

This reasoning can be extended to all reflections of the original spectrum. Therefore Equation (6) can be rewritten as

$$Y_A(\upsilon) = \sum_{k=0}^{\frac{N}{2}-1}[a_k e^{jk\pi/N} + a_{N-k}e^{j\pi(1+k/N)} + \\ + a_{N+k}e^{j\pi(-1+k/N)} + \ldots]\delta(\upsilon - k\omega_0) \quad (9)$$

where $\upsilon \in [0,\Omega]$.

After the complex function $Y_A(\upsilon)$ is decomposed in real and imaginary parts, its magnitude is obtained, as

$$|Y_A(\upsilon)| = \sum_{k=0}^{\frac{N}{2}-1}|\,a_k + \sum_{m=1}^{\infty}(-1)^m[a_{mN-k} + a_{mN+k}]\,|\delta(\upsilon - k\omega_0) \quad (10)$$

Consider now an integer delay $d_{\ell=2,m=1} = t_s$, $f(nt_s)$ can be rewritten as

$$f_B(nt_s) = \sum_{k=-\infty}^{\infty} a_k \cos\left(k\omega_0 nt_s + k\frac{2\pi}{N}\right) \quad (11)$$

for $k\omega_0 d_{l=2,m=1} = k\,2\pi/N,\ 0 \le n \le N$.

$$Y_B(\upsilon) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} a_k\,\delta(\upsilon - k\omega_0)\,e^{j\frac{k2\pi}{N}} + R_\Omega(\upsilon) \quad (12)$$

The same reasoning applied to $f_A(nt_s)$ can be done for $f_B(nt_s)$, obtaining the magnitude of $Y_B(\upsilon)$ as

$$|Y_B(\upsilon)| = \sum_{k=0}^{\frac{N}{2}-1}|\,a_k + \sum_{m=1}^{\infty}[a_{mN-k} + a_{mN+k}]\,|\delta(\upsilon - k\omega_0) \quad (13)$$

When we compare Equations (10) and (13), it is possible to observe that some of the terms, $[a_{mN-k},\ a_{mN+k}]$ for m odd, have opposite signals in each equation.

We have the coefficients $C_{FD}(k) = a_k - \sum_{m=1}^{\infty}[a_{(2m-1)N-k} + a_{(2m-1)N+k}] + \sum_{m=1}^{\infty}[a_{2mN-k} + a_{2mN+k}]$ concerning the fractional delay sampling, and $C_{ID}(k) = a_k + \sum_{m=1}^{\infty}[a_{(mN-k)} + a_{(mN+k)}]$ concerning the integer delay sampling. Now suppose that $C_{FD}(k)$ and $C_{ID}(k)$ are both positive or both negative. Comparing $C_{FD}(k)$ and $C_{ID}(k)$ we observe that this condition is true if $a_k + \sum_{m=1}^{\infty}[a_{2mN-k} + a_{2mN+k}] \ge \sum_{m=1}^{\infty}[a_{(2m-1)N-k} + a_{(2m-1)N+k}]$. Considering the condition above, we have that the average of $|Y_A(\upsilon)|$ and $|Y_B(\upsilon)|$ becomes equivalent to the magnitude of the average of $Y_A(\upsilon)$ and $Y_B(\upsilon)$ as

$$|\overline{Y}(\upsilon)| = \sum_{k=0}^{\frac{N}{2}-1}|\,a_k + \sum_{m=1}^{\infty}[a_{2mN-k} + a_{2mN+k}]\,|\delta(\upsilon - k\omega_0) \quad (14)$$

Now comparing Equations (10), (13) and (14) we can see that part of the reflected harmonics, those with coefficients a_{mN-k}, a_{mN+k}, m odd, are eliminated in $|\overline{Y}(\upsilon)|$. That occurs due to different phase shifts in the reflected harmonics for fractional and integer delays. Those results are true even for signals with high harmonic distortion, as the square and ramp waveforms, and for a small N.

It is important to observe that the most significant reflected harmonics, with frequencies in the interval $[(\omega_0(N/2),\ \omega_0(3N/2))]$ and coefficients a_{N-k}, a_{N+k}, are eliminated by a simple average.

Usually most of aliasing error energy is in the interval above, therefore in many cases we can eliminate the necessity of an anti-aliasing filter with this technique. Even if the anti-aliasing filter is still necessary, it will be much easier to design. The use of the fractional delay technique allows the anti-aliasing filter to have higher cut-off frequency and softer decay, if it is at all necessary, without the need to increase the sampling frequency.

Since the number of quantization bits is directly related with the sampling interval, for high-accuracy measurements it is important to consider a high sampling interval. The use of fractional delay, besides reducing the need for filters, allows eliminating most of the aliasing error without an increase of the sampling rate. As a consequence, it allows more quantization bits to be used.

Even if the fractional delay technique introduces twice as many data to process, the accuracy of the data acquisition system usually is a much more complex problem than data processing, that can be easily done by any computer.

3. PCA APPLIED TO HARMONIC ESTIMATION

The Principal Components Analysis (PCA), as the Fourier transform, can be defined as a linear decomposition of the original signal in specific bases. Considering this fact, PCA can also be applied to harmonic estimation of periodic signals.

Consider now the periodic signal $f(t)$ described in the previous section. After a positive zero-level crossing of this signal, N samples are acquired during a single period, in sequence, at time intervals separated by t_s seconds. After the next positive zero crossing, a time interval of d_f is waited before restarting the acquisition, obtaining that way two sample sequences of N samples, with a delay d_f between them. This process is then repeated until the acquisition of $n \geq N$ sample sequences, each delayed of d_f from the previous sequence.

A data matrix $\boldsymbol{M_f}$ is created, where each sample sequence corresponds to a different row. When PCA is applied to the data matrix $\boldsymbol{M_f}$, the obtained eigenvalues correspond to the amplitude of the harmonics of $f(t)$. If d_f is a fractional delay, the results of previous section can also be applied to PCA.

4. EXPERIMENTAL RESULTS

In order to illustrate the efficiency of the fractional delay technique and of the combined technique PCA-FD in reducing aliasing errors, both computer simulations and laboratory measurements have been done. These results are shown in the following subsections.

4.1. Simulations

Simulations were conducted using the software Scilab to simulate numerically the square waveform shown in the following equation:

$$f(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi f_0 t(2k-1))}{2k-1} \tag{15}$$

The signal $f(nt_s)$ is sampled from $f(t)$ at $f_0 = 50$ Hz, $M = N/2 = 75$, and a sampling rate $t_s = 1/(2Mf_0) \approx 133.4\ \mu$s. To illustrate the results of the Equation (14), we generate $f(nt_s)$, first without delay, and then with fractional delay $d_f = t_s/2 = 66.7\ \mu$s. We denote here $|Y(\upsilon)|$ the magnitude of the discrete Fourier transform of signal $f(nt_s)$, without delay. We also denote $|\underline{Y}(\upsilon)|$ the average of the magnitudes of the discrete Fourier transforms of signal $f(nt_s)$, generated both without delay and with the fractional delay d_f.

Consider the aliasing errors $|Y(\upsilon)|-|Y_{REF}(\upsilon)|$ for the integer delay and $|\underline{Y}(\upsilon)|-|Y_{REF}(\upsilon)|$ for the fractional delay, where $Y_{REF}(\upsilon)$ is the DFT of the reference signal $f_{REF}(nt_s)$, band-limited to the M first harmonics of $f(t)$. We can

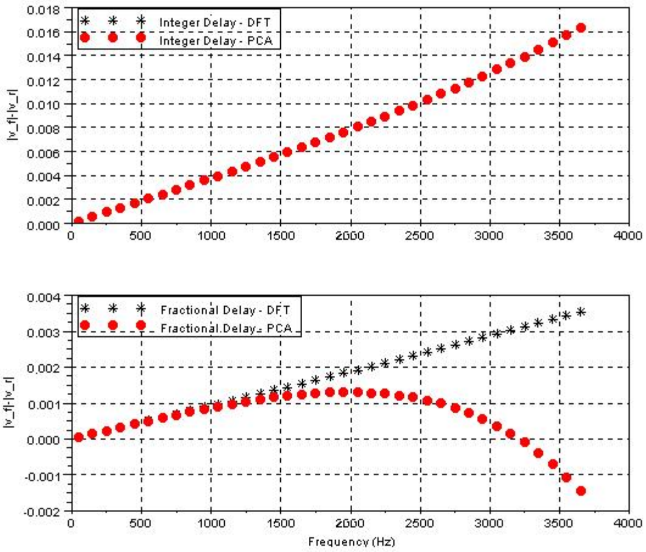

Fig. 1. Square Waveform - Simulation Results.

observe in Fig. 1 that the aliasing error for the integer delay is much higher than the aliasing error for the fractional delay. Notice that the even harmonics are all zero for this signal, so Fig. 1 shows only odd harmonics.

To illustrate now the results of section 3 we created a matrix $\boldsymbol{M_{sim}}$, considering a delay d_f, and $n = 2N$. To calculate the harmonics of $f(nt_s)$, we multiplied the matrix $\boldsymbol{M_{sim}}$ by its transpose, and then obtained the eigenvalues. We created a vector $\boldsymbol{v_{sim}}$ with the eigenvalues organized in decreasing order. Each element corresponds to a different frequency. To verify this algorithm, we also created a reference vector $\boldsymbol{v_{ref}}$, where each element $v_{ref}(k) = 4/(2k\pi)$, for $1 \leq k \leq M$. The difference between $\boldsymbol{v_{sim}}$ and $\boldsymbol{v_{ref}}$, denominated E_{sim}, is also shown in Fig. 1.

The upper part of Fig. 1 shows the previously discussed results for the integer delay $d_f = t_s$. In this case, both techniques, FD-DFT from section 2 and FD-PCA from section 3, present the same results, estimating the harmonics without reducing aliasing. The total aliasing error, in both cases, is $||E_{alias}|| = \sum_{\upsilon=1}^{N/2-1} ||Y(\upsilon)|-|Y_{REF}(\upsilon)|| \approx 0.28$. The lower part of Fig. 1 shows the results, for both PCA and DFT, for a fractional delay $d_f = t_s/2$. In this case, the algorithms have different results. Although both algorithms present a reduction of the aliasing error, PCA is more efficient, reducing the total aliasing error in approximately 90%. The total aliasing error for the fractional delay is $||E_{DFT}|| \approx 6.50 \times 10^{-2}$ for DFT and $||E_{PCA}|| \approx 3.06 \times 10^{-2}$ for PCA.

4.2. Data Acquisition

To confirm further the results obtained in the previous subsections, several laboratory measurements were done with the 8 1/2-digit multimeter Agilent 3458A operating as an ADC. The programmable waveform generator Agilent 33250A was used to synthesize the waveforms. Both equipments were controlled by a PC through GPIB ports and the Labview software. We employed the Agilent 3458A multimeter as our ADC due its stability, low-noise, and

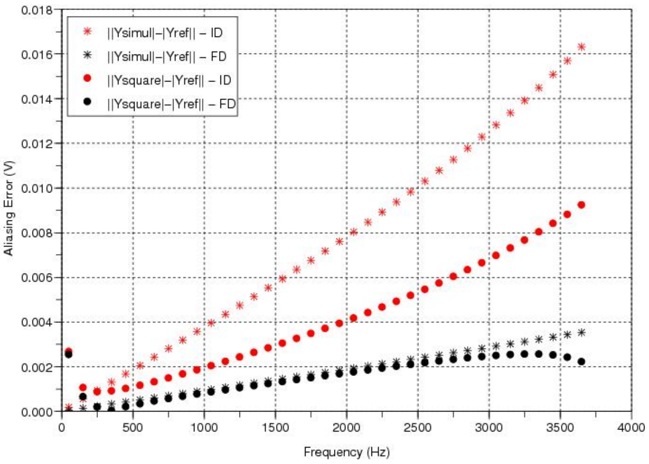

Fig. 2. Square Waveform - FD-DFT.

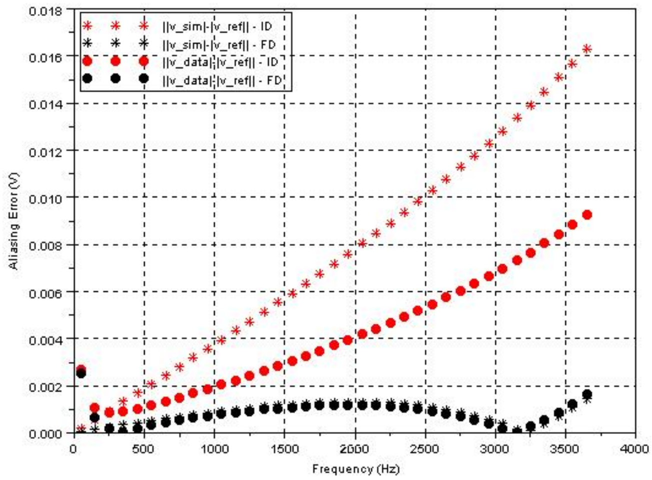

Fig. 3. Square Waveform - FD-PCA.

high sampling rate; as well as to its popularity in different areas of electrical metrology [4], [5].

We did asynchronous measurements using the multimeter internal trigger to start the data acquisition at a positive zero-level crossing. In this measurement experiment, we programmed the generator to create the square waveform described in Equation (15), considering the parameters described in the previous subsection and amplitude 1 V. For this experiment, we selected the aperture time $t_a = 100\ \mu s$, which guarantees 21 bits for quantization [6], [7].

From each cycle of the waveform we obtained a sequence of $N=2M=150$ samples. Although only two sampled sequences delayed of $d_f=t_s/2$ were necessary, we acquired $2N=300$ sampled sequences with the fractional delay between each, in order to reduce noise and gain drifts of the generator.

In Fig. 2, we compare the data aliasing errors with the simulation errors for the square waveform, considering the technique FD-DFT described in section 2. If we consider the signal sampled with an integer delay (ID), we can observe the influence of the ADC integration process, that presents the effect of a low-pass filter, [2], [6]. The total data acquisition aliasing error is $||E_{DFT\ DATA}|| \approx 0.15$. Considering now the fractional delay (FD), both simulation and data acquisition errors are approximately the same. In this case the integration process is not significant. The total data acquisition aliasing error for fractional delay is $||E_{DFT\ DATA}|| \approx 5.8 \times 10^{-2}$, a reduction of 80% in comparison to the original aliasing error.

We also applied the combined technique and PCA-FD to data acquisition. We used 2N = 300 sampled sequences to generate de data matrix $\boldsymbol{M}_{data}$, and then to calculate the eigenvalue vector $\boldsymbol{v}_{data}$. The difference between $\boldsymbol{v}_{data}$ and the reference vector $\boldsymbol{v}_{ref}$, denominated E_{data}, is shown in Fig. 3, for both an integer delay (ID) $d_f = t_s$ and a fractional delay (FD) $d_f = t_s/2$. In this figure a comparison between the data acquisition error E_{data} and the simulation error E_{sim} is also presented.

Here also we can observe the influence of the ADC integration process. When we consider the integer delay the total data acquisition aliasing error is $||E_{PCA\ DATA}|| \approx 0.15$. Considering now the fractional delay, the total data acquisition aliasing error is $||E_{PCA\ DATA}|| \approx 3.04 \times 10^{-2}$, a reduction of 90% in comparison to the original aliasing error.

It is important to observe that, if instead of fractional delay we had sampled our signal with a sampling rate twice as larger, we would loose several bits of resolution. Besides that, with fractional delay (FD-DFT) we eliminated the aliasing up to the $3M^{th}$ harmonic, and with the higher sampling rate we would only eliminate the aliasing up to the $2M^{th}$ harmonic.

5. CONCLUSIONS

In this paper we applied the fractional delay sampling technique to estimate harmonics in asynchronous sampled signals. We also combined this fractional-delay technique with PCA. Both techniques allow a significant reduction of aliasing error, at the same time avoiding the need of high sampling rates and distortions of the original signal. The efficiency of these techniques was verified by both simulations and laboratory measurements.

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