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HOLOGRAPHIC PRISM – THE NEW PLANE ANGLE MEASURE ON BASE OF HOLOGRAM ARRAY IN CRYSTALLINE PHOTOCHROMIC NANO-MATERIAL

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Abstract – A new multivalued measure of plane angle is proposed, which is based on a highly stable system of superimposed holograms recorded in a photochromic material. Their mutual position forms a reference set of angles, which is stored by the measure and can be reproduced by a reference laser. The methods for fabricating a holographic measure (recording and reproducing holograms forming it) are discussed. One of two possible modifications of such a measure is implemented on the basis of calcium fluoride crystals with color centres. The technique for preparing this measure (holographic prism) and its properties are described. Based on this prism, one can develop a new generation of angle-measuring or -setting instruments that simultaneously satisfy two contradictory requirements, i.e., mobility and high discreteness and accuracy of angular measurements.

Keywords: plane angle measure, holographic prism, nano-material.

1. INTRODUCTION

Plane angle is a unique physical quantity, which has a natural absolute reference, i.e., total angle. The principle of angular measurements is in division of this value [1]. This division is generally performed using a multivalued angle measure, which is applied to calibrate circular scales of rotational devices. A conventional measure is a regular polyhedral fused silica prism, whose angles are set by normals to its faces [2]. Each normal is implemented physically by the autocollimator axis when the cross hairs in its focal plane are aligned with the image arising as a result of collimated beam reflection from the prism face. The angle between two normals is reproduced by rotating the prism around the axis perpendicular to the autocollimator measuring plane. Thus, the set of angles stored by the prism is determined by the mutual position of its faces and is reproduced using light beam reflection from them and sample rotation. The prism reproduces angles with effective values close to m·(360/n)°, where *n* is the number of lateral faces and m = 1, 2, n–1 (actually, n ≤ 72). These values can be determined by preliminarily calibrating the prism with an error from several arc seconds to several tenths of arc seconds; this error is related to methodical and instrumental components and the nonplanarity and piramidality of the prism faces. After calibration, the set of prism angles is used to calibrate the circular scale of a rotational device on which it is installed.

A quartz prism has the following drawbacks: large weight and size; discreteness of the formed plane angle circular scale, which is limited by the number of faces; low-production efficiency (prism preparation is in principle a custom-made, time-consuming process); and the hazard of spontaneous sharp changes in the optical and geometric characteristics (so-called devitrification).

In view of the aforesaid, only a limited number of institutions responsible for monitoring angle measuring or setting instruments in stationary systems are equipped with quartz prisms. At the same time, there are ranges of applications of these instruments where on-line field monitoring of their accuracy and stability is extremely desirable, e.g., navigation; geodesy in distant regions; building and mining; robotics; and the operation of space, deep-water, and well equipment.

We propose a new multivalued plane angle measure based on the holography principles [3]: a holographic prism (see Fig. 1), which has a number of significant advantages over the quartz prism [4].

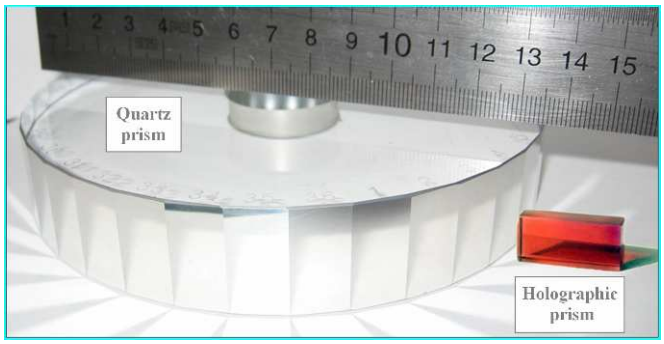

Fig. 1. Holographic prism (right) and 36-face fused silica prism (left).

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2. PRINCIPLE OF HOLOGRAPHIC PRISM OPERATION

The proposed optical element is a parallelepiped made of a photochromic material (referred to as sample below), in which a system of superimposed holograms is recorded. Their mutual spatial position forms a set of angles (multivalued holographic measure) stored by this element. The exposure of this sample to a reference laser beam induces a response in the form of several diffracted beams. Depending on the recording method, they arise successively or simultaneously upon rotating the sample and cover a limited range of angles; these beams are recorded by a photoelectric detector. The rotation of the sample makes it possible to form a full angular scale.

The angles between the directions set by the holograms are functional analogs of the angles between the quartz prism normals. Hence, this element can be referred to as a holographic prism. For this prism, the angle between the holograms forming it can be fairly small (on the order of an arc minute). This circumstance provides a high discreteness of the realized circular scale and, correspondingly, high accuracy of angular measurements.

A holographic prism, as was noted above, can be implemented in two modifications. In the first modification (I), a reference beam and diffracted beams, which occur successively during prism rotation, are located in the same plane oriented perpendicularly to the axis of rotation. In this case, each diffracted beam propagates in the same fixed direction. In the other modification (II), the reference beam is oriented perpendicularly to the plane in which the fan of diffracted beams lies. All holograms of the system are simultaneously involved in the formation of these beams. For both modifications, the set of angles reproduced by the holographic prism is set by the beams diffracted from individual holograms.

The principles of holographic prism implementation in modifications I and II are presented in Figs. 2, 3, respectively. In both cases, the system based on this prism contains the same set of main elements, i.e., a laser *1,* which supplies reference beam 2 to a holographic prism *3,* which is mounted on a rotational table *4* and a detector *6,* which records diffracted beam 5.

In modification I, when a hologram is recorded in the prism *3,* the reference beam 2 and diffracted beam 5 lie in the same plane. In this case, the angle of recording beam convergence in the crystal bulk is chosen proceeding from the specific conditions of prism operation (working beam diameter, focal length of the objective for recording diffracted beams, etc.). A system of holograms is formed by their successive recording in the same volume. After recording each hologram, the sample is rotated by a specified angle around the axis oriented perpendicularly to the recording-beam convergence plane.

When the thus formed holographic prism *3* is used as an angle measure, it is installed on the rotational device *4;* in this case, the reference laser beam 2 is directed perpendicularly to the rotation axis of this system (Fig. 2). A laser with any wavelength can be used as the reference laser *1*. Each hologram recorded in the prism system generates a diffracted beam (5) if its orientation with respect to the reference beam 2 satisfies the Bragg condition[1]. All holograms of the system successively response in the same direction but at different positions of the prism, determined by the angle of its rotation with respect to the rotation axis. Thus, the fixation of a current diffracted beam (6) at the same point of the immobile indicator 5 as for the previous beam indicates the rotation of the sample by one of the angles specified by the spatial mutual position of holograms in the system. The values of the angles are set during recording a prism and are refined in its calibration.

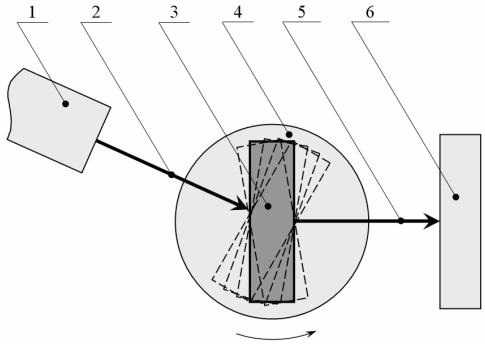

Fig. 2. Holographic prism in modification I (top view): (1) reference laser, (2) laser beam, (3) holographic prism, (4) rotational table, (5) radiation detector, and (6) diffracted beam.

Holograms can be recorded by oppositely directed beams of two reference lasers. In this case, the data set on the crystal positions in which the Bragg condition is satisfied is formed by a set of oppositely directed beams diffracted from all holograms of the system and recorded by two photo-detectors. This set makes it possible to minimize the errors caused by the radial beatings of the rotational device and the error in positioning the prism on it. A plane mirror reflecting the reference beam transmitted through the sample in the opposite direction can play the role of the second laser.

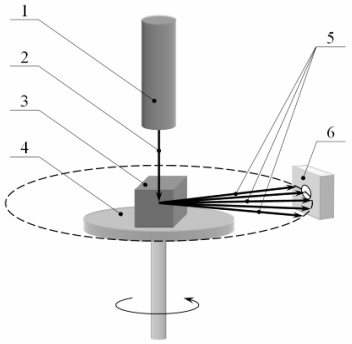

Fig. 3. Holographic prism in modification II (top view). Designations are the same as in Fig. 2.

In modification II, the reference beam 2 is directed along the table rotation axis and perpendicularly to the propagation plane of diffracted beam. The sample rotation axis during recording should lie in the recording-beam

[1] It should be taken into account that, due to the light refraction in the sample, the angle between the reference and diffracted beams in air differs from the doubled Bragg angle in the crystal.

convergence plane. Their convergence angle should be set properly to ensure orthogonality of the diffracted beam 5 and the rotation axis. In turn, the recording-beam convergence angle is determined by the ratio of the wavelengths of the reference laser *1* and the recording laser wavelength. As was noted above, in contrast to modification I, the reference beam 2 induces simultaneous response of all holograms of the system. Using a prism of modification II, one can fix angles as in modification I by rotating the sample up to that instant when the current diffracted beam (5) takes the position of the previous beam before rotation. An alternative version is possible when all beams are simultaneously fixed on the scanning scale of the reference device *6,* which is calibrated in angular units (Fig. 4).

In modification II, as it is shown in Fig. 4, in the presence of the second, oppositely directed reference laser (or reflecting mirror) automatically provides formation of two fans of diffracted beams.

In this paper, we describe the fabrication technique and characteristics of a holographic prism in modification I.

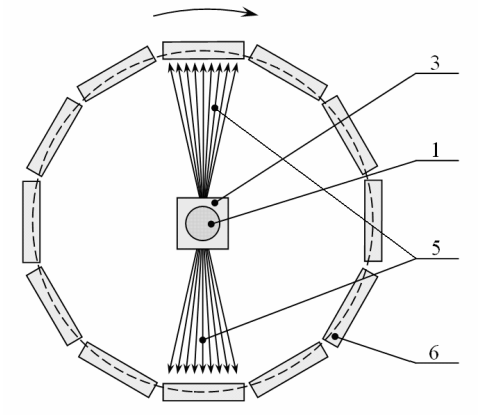

Fig 4. Holographic prism in modification II: detection of the diffraction signal using the circular scanning scale of the detectors, in the presence of two fans of diffracted beams. Designations are the same as in Fig. 2.

3. HOLOGRAPHIC GRATINGS RECORDING

It is natural to use crystals (most stable photochromic materials) as matrices to record a system of superimposed holograms forming a holographic prism. We chose calcium fluoride (CaF_2) crystals (fluorite mineral) with color centres as these matrices. Color centres are formed in ionic crystals as a result of calcination in a reducing atmosphere of metal vapors (so-called additive coloring). During this procedure, chemical reactions occur on the crystal surface, which give rise to diffusion of two fluxes (anion vacancies and electrons) into the crystal bulk. Color centres are formed as a result of their recombination. The photochromism of additively colored CaF_2 crystals is based on the processes of photoinduced inter-transformation of color centres [5, 6]. These processes occur at temperatures T exceeding 50 °C.

We used pure (undoped) fluorite crystals and crystals with a low - $(10^{17}–10^{18})$ cm^{-3} - concentration of alkaline impurity (sodium)[2]. Coloring was performed in a system known as a heat pipe, which makes it possible to almost independently control the main parameters of the process, i.e., the sample temperature and metal vapor pressure [7, 8]. Our technique of additive coloring made it possible for the first time to uniformly color CaF_2 crystals with sizes limited by only the sample container size (a cylindrical container 25 mm in diameter and 70 mm in height was used). The coloring of fluorite crystals with thicknesses not exceeding 0.5 mm was previously reported on.

Additive coloring of CaF_2 crystals by this technique led to the formation of simple color centres in them, containing one to four anion vacancies with electrons (respectively, *F, M, R,* and *N* centres) and colloidal centres (see Table 1).

Table 1. Main characteristics of colour centres

Centre kind	Vacancy number in a centre	Spectral range, nm	Centre size, nm
Simple (*F, M, R, N*)	1–4	200–550	0.3–0.5
Colloidal	10^4–10^5	550–600	10–30

The absorption bands of simple color centres are in the visible and UV spectral regions *(X* < 550 nm; Fig. 5).

Colloidal centres are nanoscale inclusions of metallic calcium particles [8][3]. In the absorption spectrum of CaF_2 crystals, the band peaking in the wavelength range of (550–600) nm, depending on the average size of colloidal particles ((10–30) nm [5]), is due to colloidal centres. This band is much wider than those of simple centres.

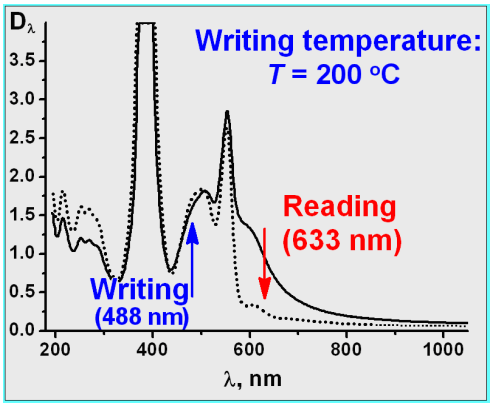

Fig. 5. Absorption spectra of (solid line) an additively colored CaF_2 sample and (dotted line) the same sample with a hologram recorded by 488-nm argon laser radiation at $T \sim 200$ °C.

[2] The alkaline impurity concentration in pure crystals is at the level $N_{Na} = (10^{14}–10^{15})$ cm^{-3}.

[3] In undoped and lightly doped (with alkaline impurities) CaF_2 crystals, colored by this technique, so-called quasi-colloidal color centres are absent. Apparently, these centres are more or less large associates of *F* centres. Their absorption bands cover a wide spectral range from ~550 nm to 2 μm in pure crystals; in crystals doped with alkaline impurities, this range expands to 3.5 μm.

The color centres formed in pure fluorite crystals and in crystals with low contents of alkaline impurities are identical, as well as the processes of their photoinduced transformation. Small additives of alkali metals increase the photosensitivity of the crystal because their incorporation into crystals is accompanied by the formation of an additional (with respect to the coloring procedure) number of anion vacancies, which play an important role in the photoconversion of color centres.

At the same time, alkali impurities with concentrations exceeding 10^{18} cm[4] decrease the hologram diffraction efficiency, reason being that they stabilize color centres and hinder their spatial redistribution in the interference field of recording radiation, which is the underlying process in hologram recording (see below).

The mechanism of recording effective holograms in ionic crystals with color centres has a diffusion-drift character [9–11][4]. During recording at $T > 50$ °C, color centres leave antinodes of the interference pattern and are concentrated in its nodes. In this case, it is not centres that move, but rather their components: anion vacancies and electrons formed upon photoionization of color centres. Vacancies and electrons recombine in the nodes of the interference pattern. The type of color centres formed in this case is determined by the types of the centres present in the colored sample (their absorption spectrum), the recording laser wavelength, and the crystal temperature. As a result of hologram recording, centres of the same type (simple, quasi-colloidal, or colloidal) are formed in the crystal. The spatial change in the optical constants that form the holographic grating is determined by modulation of the concentration of these centres [10, 12]. The diffusion-drift recording mechanism leads to a significant difference in the holographic grating and interference field profiles. The nonlinearity of recording leads to the appearance of several diffraction orders and impossibility of re-exposing the hologram. An increase in exposure mainly increases the above-mentioned difference; photoerasure of a hologram even at $T \sim 200$ °C requires an extremely large exposure [11, 13].

When a prism was fabricated in modification I, holograms were recorded by two methods. One of them is based on the photochemical reaction of transformation of simple color centres into colloidal ones, which occurs under argon laser radiation at a wavelength of 364 nm[5] at $T = 200$ °C. In this case, an amplitude-phase hologram is formed at the reading wavelength used (632.8 nm) [13].

The second method is based on the reverse reaction. It occurs under 488-nm argon laser radiation (Fig. 5); this radiation shifts equilibrium in the colloidal-simple centres system to the latter. Recording was performed at the same temperature. In this recording technique, a phase hologram with an insignificant amplitude contribution is mainly formed at the reading wavelength [11]. The diffraction efficiency of the holograms recorded by both methods depends on the degree of crystal coloring and the recording parameters. In both methods, it reached 15 % for single holograms.

The optical scheme of the interferometer for recording holograms, beginning with the beam splitter, was placed in a thermostated housing equipped with a heater and a thermocouple. The feedback circuit of the heater power supply maintained a temperature of about 200 °C in the housing with an error of 0.1 °C.

The sample in the housing was mounted on a rod installed on the rotational device. The sample was rotated after recording each hologram (see above). Holograms were recorded in copropagating beams. We describe below the recording procedure and report the characteristics of a holographic prism in modification I, recorded by an UV argon laser in a CaF_2:Na sample ($N_{Na} = 4.7 \times 10^{17}$ cm^{-3}) with sizes of 1.40 x 0.85 x 0.77 cm. The prism has a volume of 0.9 cm^3 and a weight of 2.9 g.

When the first hologram of the system was recorded, nearly all simple centers were transformed into colloidal color centers and concentrated in the interference field nodes. When recording subsequent holograms, the sample was rotated and these nodes shifted with respect to the crystallographic directions. As a result, the colloidal centres were partially redistributed in the sample volume. Thus, the diffraction efficiency of each subsequent hologram was formed due to attenuation of the previously recorded holograms.

The system of five holograms with a Bragg angle of 10° was recorded in the prism. After recording each hologram, the crystal was rotated by an angle of (15.1 ± 0.1)°. The initial sample position on the rotational table was chosen so as to achieve the recording of hologram no. 3 according to the symmetric scheme (the sample surface was perpendicular to the angle bisector between two interfering beams).

4. HOLOGRAPHIC PRISM CHARACTERISTICS

After recording, the holographic prism was installed on the rotational table (Fig. 2). In this case, the angle between the reference laser-prism and prism-photodetector directions was 20°.

Fig. 6 shows the dependence of the diffracted light intensity on the angle of prism rotation, and presents the profile of angular selectivity of one of the holograms. The significant sample thickness in the reference beam propagation direction (1.4 cm) provides narrowness of this profile, whose shape is close to Gaussian. The peak position (profile centroid), determining one of the reference points of the angle measure, can be found with an error much smaller than the profile half-width (1.8').

An increase in the sample temperature leads to equalization of the colloidal center concentration over the sample volume; i.e., the hologram decays. However, this decay only occurs upon strong crystal heating.

The sample was subjected to cyclic 72-h annealing in the temperature range of $T = (150–350)$ °C. After each annealing, the sample was cooled to room temperature, and the hologram diffraction efficiency was measured. Then, the

[4] This holds true for holograms with a diffraction efficiency of several percent or several tens percent. Recording of holograms with relatively low diffraction efficiency has a local character.

[5] All simple color centers have absorption bands at wavelengths close to this wavelength.

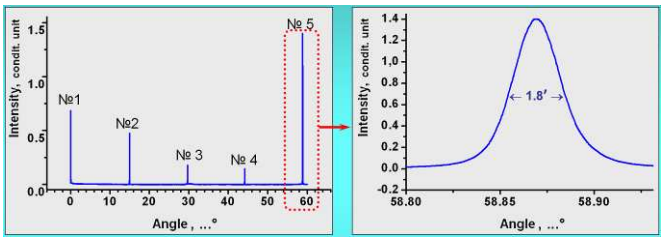

Fig. 6. Dependence of the diffracted light intensity on the angle of holographic prism rotation in modification I, recorded in a CaF_2:Na sample ($N_{Na} = 4.7 \times 10^{17}$ cm^{-3}); the angular scale zero is determined by the rotational table position at which the reference corresponding to hologram no. 1 is maximum (left) and angular selectivity profile of hologram no. 5 (right).

next annealing-measurement cycle was carried out at higher annealing temperatures. It can be seen from the table that the hologram thermally decays at temperatures exceeding 200 °C. The holograms recorded by means of the colloidal-simple color center transformation turned out to have the same thermal stability. Illumination of the crystal in the short-wavelength spectral range, where strong absorption occurs ($k < 550$ nm), may decrease the hologram decay temperature, but only at a fairly high radiation power, i.e., several tenths of a watt per square centimeter. Since holographic prism operation implies the use of a low-power reference laser, it is obvious that its service life is almost unlimited[6].

Thus, a hologram recorded on color centres in a fluorite crystal is retained even under long-term heating of the sample to ~200 °C. At the same time, the thermal expansion of the crystal lattice modifies the Bragg angle of the hologram. For the holograms that are asymmetric with respect to the sample plane, along with the above-mentioned factor, it is also necessary to take into account the temperature dependence of the refractive index of the crystal, which determines the change in the angle between individual holograms of the system. Both of these effects manifest themselves at fairly small variations in the crystal temperature. In the linear approximation, the temperature dependence of the holographic grating period has the form

$$h(T) = h_0(1 + \alpha\Delta T), \qquad (1)$$

where h_0 is the period corresponding to some initial temperature, α is the thermal expansion coefficient, and ΔT is the deviation of the crystal temperature from initial (in this calculation, it was assumed to be 20 °C).

When a hologram is read by radiation with the wavelength λ' differing from the recording wavelength, the angle between the reference and diffracted beams is

$$\varphi'(T) = 2\arcsin\left(\frac{n\lambda'}{2h(T)}\right) = 2\arcsin\left(\frac{n\lambda'}{2h_0(1+\alpha\cdot\Delta T)}\right) \cong \varphi_0'(1 - B\cdot\Delta T) \qquad (2)$$

where φ_0' is the angle corresponding to the initial temperature. Thus, for small variations in temperature, the Bragg angle, as well as the angle between the reference and diffracted beams, is a linear function of change in the crystal temperature.

To determine the coefficient B, it is necessary to analyze the refraction of the interfering (reference and diffracted) beams at the input and output of the sample. This analysis shows that to reduce the instability of the measure under consideration no a level not exceeding 0.1″, it is necessary to stabilize temperature with an error of about 0.1 °C. The well-known analytical dependence (5) makes it possible to take into account, using a holographic prism, the change in the angle φ' with temperature in the program processing of prism signals. Therefore, the requirements for the level of prism thermal stabilization can be reduced to about a Celsius degree if a precise (0.1 °C) thermometer is used.

Since the difference between the coefficients B for symmetric and asymmetric holograms does not exceed 0.01″/°C, these deviations, determining the temperature change in the angle between individual holograms, can be neglected in signal processing.

5. METROLOGICAL ENSURING

Metrological ensuring of the holographic prism development process depends essentially from what system of holograms, respectively, what kind of the holographic prism, we are dealing with. It should be noted that the second kind of holographic prism is an *active* many-value angle measure, contrary to reflection prism and holographic prism of the first kind.

Metrological assurance of the holographic prism making includes generally three basic tasks: (i) choice of appropriate nominal reproducible angles, (ii) choice of other prism metrological characteristics and normalisation of them, and (iii) calibration of the prism [14, 15]. Approaches to the task solutions are discussed below.

(i) Effective using of the holographic prism undermines that it ensures an optimal set of base (reference) angles kept and reproduced by it. The choice of these angles directly determines the calibration spacing and redundancy of derivable data. The principles of the reference angles choice are developed. In the perspective it is supposed to install the holographic prism in set-ups equipped with angle sensors to ensure an opportunity of their self-checking.

From 5 to 9 superimposed holograms with constant and variable angular step in the range 30′ – 15° are written in embodied specimens.

(ii) Holographic prism metrological characteristics under normalization could be separated in three groups:

– characteristics representing reciprocal space location of holograms written in the crystal in a special co-ordinate system which is determines the measurement plane;
– parameters characterizing crystal orientation in the above-mentioned co-ordinate system;
– characteristics of crystal form imperfections.

(iii) The technique for execution measurements at the mutual calibration of circle scales using the above set of

[6] 20 years ago, we recorded holograms on quasi-colloidal centres in additively colored fluorite crystals. To date, they have not undergone any significant changes.

angles including algorithms of data treatment and their program realization is advanced [15].

The calibration procedure is a set of redundant joint (combinatorial) measurements (see Fig. 7). During its fulfilment various combinations of angular intervals of two circular scales are measured. The latter are: the basic scale formed by holograms, and the auxiliary scale of rotating apparatus used. Anybody get required combinations by alternate fulfillment of two rotary movements: joint single-step rotation of constrained bearers (rotors), and angular shift of rotors relative each other which is multiple to calibration step.

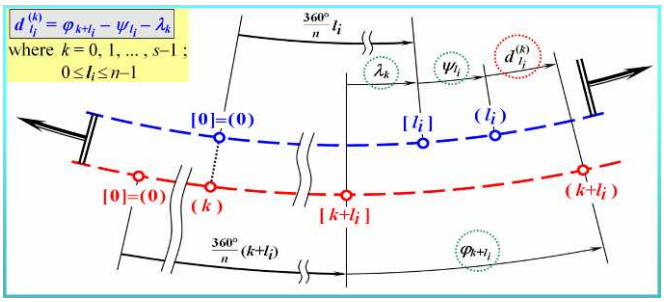

Fig. 7. Forming of coupling equations for combined parameters in the procedure of holographic prism calibration.

In Fig. 7, parts of two superposed circular scales under calibration are shown with dashed arc, and following notations are introduced:

$d^{(k)}_{l_i}$ – reading in k-*th* series, l_i-*th* position of constrained rotors, which is equal to a difference of readings on the first and the second scales (where $l_i \equiv i$ if both of the scales are full without skipped marks);

$\{\varphi_j, \psi_j\}$ – desired calibration parameters, i. e., actual mark deflections of two scales from nominal one which lie uniformly on a circle; for partial (not full) circular scales some actual marks are possible to be left out;

$\{\lambda_k\}$ – residual deflections of relative angular shifts of rotors from formulary values which are multiple of a calibration step;

k – series order number;

i – reading order number (within series);

[…] – order numbers of nominal scale marks;

(…) – order numbers of actual scale marks.

For details see [15].

6. CONCLUSIONS

The holographic prism may have substantial preferences compared to reflection prism:

(i) it allows rendering automatic the registration of measurement results and keeping these results on the external information carrier;

(ii) it allows realizing the non-uniform array of directions that may ensure the very small calibration step;

(iii) the prism in modification II as self-contained device (coupled with referent laser) is an active multivalued angle measure;

(iv) a goniometrical device using this prism as a built-in component can be compact, which may produce a new quality including self-checking feature;

(v) smaller dimension of the device make it less sensitive to vibrations, which lower an exactness of angle measurements;

(vi) the reflection fused quartz prism endures very slow but aging accompanying modification of its configuration whereas many crystals are stable in geological time scale;

(vii) production of reflection prisms is a piece-work and needs significant time and financial expenses whereas fabrication of holographic prisms is much more economical (at relatively high initial expenses for deriving a new technology).

REFERENCES

[1] V. A. Granovsky, M. D. Kudryavtsev, "The plane angle concept and its unit in the context of traceability problem", *Proc. XVIII IMEKO World Congress*, Rio de Janeiro, Brazil, Sept. 2006 (http://www.imeko.org).

[2] M. G. Boguslavsky, V. M. Eliashberg, E. E. Sharova, L. I. Fedotova, "State primary angle standard – radian", *Measurement technique* [*Izmeritel'naya Tekhnika*, in Russian], No. 7, pp. 9-10, 1972.

[3] V. A. Granovsky, M. D. Kudryavtsev, A. I. Ryskin, A. S. Shcheulin, "A new optical element – holographic prism. I. Principle of operation and experimental disposal", *Optics and Spectroscopy*, **106** (5), 774 (2009).

[4] V. A. Granovsky, M. D. Kudryavtsev, A. I. Ryskin, A. S. Shcheulin, Russian Federation patent № 2006140710, priority of invention from 14.10.2006.

[5] V. M. Orera and R. Alkala, Phys. Status Solidi A **44** (2), 717 (1977).

[6] A. S. Shcheulin, A. K. Kupchikov, and A. I. Ryskin, *Optics and Spectroscopy* **103** (3), 496 (2007) [*Optics and Spectroscopy* **103,** 507 (2007)].

[7] C. Z. van Doorn, Philips Res. Rep., No. 4, 1 (1962).

[8] A. S. Shcheulin, T. S. Semenova, L. F. Koryakina, M. A. Petrova, A. K. Kupchikov, and A. I. Ryskin, *Optics and Spectroscopy* **103** (4), 673 (2007) [*Optics and Spectroscopy* **103,** 660 (2007)].

[9] *Crystals with the Fluorite Structure,* Ed. by W. Hayes (Clarendon, Oxford, 1974).

[10] V. M. Belous, V. E. Mandel', A. Yu. Popov, and A. V. Tyurin, *Optics and Spectroscopy* 87 (2), 327 (1999) [*Optics and Spectroscopy* 87, 305 (1999)].

[11] A. S. Shcheulin, A. V. Koklyushkin, E. V. Tsygankova, and A. I. Ryskin, *Optics and Spectroscopy* **104** (6), 1028 (2008) [*Optics and Spectroscopy* **104,** # (2008)].

[12] A. S. Shcheulin, A. K. Kupchikov, A. E. Angervaks, and A. I. Ryskin, *Optics and Spectroscopy* **103** (4), 664 (2007) [*Optics and Spectroscopy* **103,** 651 (2007)].

[13] A. S. Shcheulin, A. V. Veniaminov, Yu. L. Korzinin, A. E. Angervaks, and A. I. Ryskin, *Optics and Spectroscopy* **103** (4), 668 (2007) [*Optics and Spectroscopy* **103,** 655 (2007)].

[14] A. H. Cook, "The calibration of circular scales and precision polygons", *Brit. J. appl. Phys.*, vol. 5, pp. 367-371, Oct. 1954.

[15] O. B. Basun, V. A. Granovsky, M. D. Kudrjavtsev, "Precision scales of plane angle: principles and methods of calibration", *Proc. XVIII IMEKO World Congress*, Rio de Janeiro, Brazil, Sept. 2006 (http://www.imeko.org).