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ANALYSIS OF MEASUREMENT UNCERTAINTY IN THE PROCEDURE OF GROOVE DEPTH MEASUREMENT

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Abstract – This paper presents summary of influencing quantities in the procedure of measurement groove depth *d* by use of Epival-Interphako interferometric microscope. The expression of mathematical model describing the dependence of measured depth of groove *d* in relationship with all of the influencing components has been presented.

Influencing quantities have been analyzed, and the magnitudes of contribution of those quantities that are considered` to have the largest influence on uncertainty of the result have been calculated.

This analysis has been done based on guidelines for calculation of measurement uncertainty: ISO Guide to the Expression of Uncertainty in Measurement (GUM) and Supplement 1 to the GUM – Propagation of distribution using a Monte Carlo method.

Keywords: measurement uncertainty, interferometric microscope, Monte Carlo method

1. INTRODUCTION

In the Laboratory for precise measurement of length (LFSB) for a number of year's roughness parameter: depth of groove *d* is measured by use of contact method, assuring traceability by means of National Standards of Roughness.

Recently measurements of groove depth on Epival-Interphako interferometric microscope have been reactivated. In order to improve measurement accuracy and achieve more precise results retrofit of interferometric microscope Epival-Interphako has been done.

In the development of the measuring methods for the LFSB it is highly important to provide such conditions that will insure traceability from the Primary standard of length i.e. stabilized laser (produced by NPL), as well as to reduce measurement uncertainty. In order to meet above specified requests Epival-Interphako Interferometric microscope has been upgraded, first with CCD camera and later with a change of used light source from halogen lamp to laser.

2. SHEARING PRINCIPLE OF IMAGE SEPARATION

Epival-Interphako interferometric microscope uses Shearing principle of image separation (Fig. 1).

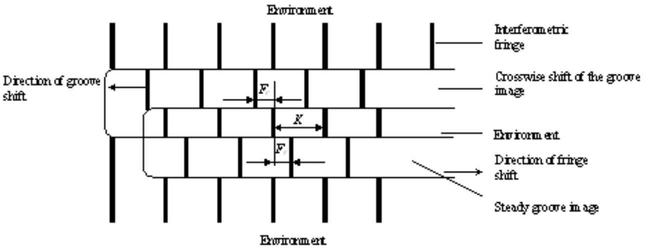

Fig. 1. Crosswise shift of the groove image obtained by Shearing principle of image separation

The environment represents the upper part of the gilded measurement surface with visible interference fringes. The object represents partial images of the gilded groove on which the interference fringes are shifted in equal opposite directions depending of the depth of groove.

3. ANALYSIS OF INFLUENCING PARAMETERS

There is a series of quantities that have influence on the uncertainty of depth of groove measurement result. Considering Shearing principle of image separation and by introducing all influencing quantities in the procedure of groove depth measurement by use of interferometric method, mathematical model has been presented:

$$d_e = (\kappa + F_r)\frac{\lambda}{2n_1} + \theta_e \cdot \alpha_e \cdot d + \delta L_{\Omega} \qquad (1)$$

where:

d_e	- groove depth of roughness standard at 20 °C
d	- nominal value of groove depth
K	- integer part of the real numbers
F_{ri}	- friction part of the real numbers
λ	- wavelength of used light source in vacuum
n_1	- refraction index of air
$\Theta_e=(t_e\text{-}20^{\circ}C)$	- deviation of roughness standard temperature from 20 °C
α_e	- linear thermal expansion coefficient of roughness standard
δL_{Ω}	- influence of the aperture

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3.1 Uncertainty of groove depth measurements using method of exact fractions $u(d_{fit})$

Image analysis was done by use of software developed in LFSB. The program was written in Visual Basic and functions in such way that image taken by camera converts into gray scale image. By use of x-y coordinates, boundaries in the form of rectangle are being defined around the interference fringe. Within set boundaries all the pixels are transmitted into a black&white combination in relation to priory set edge detectors. Arithmetic algorithms calculate the position of the fringe central line

The groove depth determined by means of exact fractions method d_{fit} can be expressed as:

$$d_{fit} = \frac{(\kappa + F_r)\cdot\lambda}{2\cdot n_1} \tag{2}$$

Based on nominal value of groove depth d and known value of used light source λ a integer part of the fringe κ has been determined, consequently $u(\kappa_i) = 0$. Influencing quantities in procedure of groove depth measurement using method of exact fractions are: friction part of the real numbers F_r, wavelength of used light source in vacuum λ and refraction index of air n_1 along with related measurement uncertainties $u(F_r)$, $u(\lambda)$, $u(n_1)$.

Standard uncertainty of the exact fractions measurement $u(F_r)$

For measurement of the exact fractions F_r standard uncertainty $u(F_r)$ has been determined out of thirty repeated measurements. For standards with nominal values of groove depth 100 nm and 4000 nm standard uncertainties are:

100 nm	$u(F\text{r})$=0,007 fringe
4000 nm	$u(F\text{r})$=0,046 fringe

Standard uncertainty of used light source wavelength $u_c(\lambda)$

Used He-Ne laser is a product of Renishaw, laser ML-10. According to Renishaw Certificate No. RR01/103601R standard uncertainty for calibration of laser equals $0,005\cdot10^{-6}\lambda$. Average annual laser frequency shift is 5 Mhz or $0,01\cdot10^{-6}\lambda$. Combined standard measurement uncertainty of used light source wavelength in vacuum equals:

$$u_c(\lambda) = 0{,}011\cdot10^{-6}\cdot\lambda \tag{3}$$

Standard uncertainty of air refraction index $u(n_1)$

In LFSB, as well as in most of the laboratories, refraction index is determined indirectly measuring parameters that have influence on air density, using modified version of Edlen equation. Moreover, uncertainty of air contexture is taken in consideration due to significant deviations from the standard especially concerning percentage of CO_2.

Value of standard uncertainty for air refraction index $u(n_1)$ has been determined and it equals $12,9\cdot10^{-8}$. Regarding groove depth measurements this value is considered to have negligible influence, therefore:

$$u_c(n_1) = negligible \tag{4}$$

Combined standard uncertainty of exact fractions method $u_c(d_{\text{fit}})$ equals:

$$u_c(d_{fit}) = 2{,}1\ nm + 0{,}003\cdot d\text{ , for } d \text{ in nm} \tag{5}$$

3.2 Uncertainty of temperature $u(\delta d_t)$

If temperature during measurements is different from referent temperature, temperature correction must be applied, according to the expression:

$$\delta d_t = \theta_e\alpha_e d \tag{6}$$

where:

d	- nominal value of groove depth
$\theta_e = t_e$ - 20 °C	- temperature deviation from 20 °C of roughness standard
t_e	- temperature of roughness standard
α_e	- linear thermal expansion coefficients of roughness standard

Since d is the nominal value of groove depth, standard uncertainty is $u(d)$=0. Therefore, uncertainty of temperature correction consists of uncertainty of measurement of standards temperature $u(\theta_e)$ and uncertainty of linear thermal expansion coefficients $u(\alpha_e)$.

Standard uncertainty of the roughness standard temperature $u(\theta_e)$

Measurements is to be performed if the deviation of roughness standard temperature does not excide ± 0,3 °C, accordingly standard uncertainty of the roughness standard temperature is:

$$u(\theta_e) = 0{,}0173\ ^oC \tag{7}$$

Standard uncertainty of linear thermal expansion coefficient $u(\alpha_e)$

The basic material for measured roughness standard is silicon mono-crystal, processed by planar technology, by means of which the rectangular grooves in the SiO_2 layer were obtained. Groove depth was measured on gilded part of measurement surface.

Since the layer of gold is around 50 nm, both in the groove and on the environment, the influence of the thermal expansion coefficient of a gold is assumed to have negligible influence on temperature deviation. Thermal expansion coefficients for silicon mono-crystal equals $2,5\cdot10^{-6}$, and for SiO_2 $0,5\cdot0^{-6}$, assuming rectangular distribution. Standard uncertainty of linear thermal expansion coefficient equals:

$$u(\alpha_e) = 0{,}577\cdot10^{-6}\quad K^{-1} \tag{8}$$

Combined standard uncertainty of thermal effects method $u_c(\delta d_t)$ equals:

$$u_c(\delta d_t) = 0{,}466\cdot10^{-6}\cdot d\text{ , for } d \text{ in nm} \tag{9}$$

3.3 Uncertainty of the aperture u(δd_{Ω})

In Epival-Interphako interferometric microscope, aperture is precisely mounted on the way of interferometer optical axis. As a result, the beam of light is perpendicular to the groove and the environment; therefore correction due to aperture influence is equal to 0.

Measurement uncertainty of the aperture $u(\delta d_{\Omega})$ depends on focus length f_k and dimensions of the aperture. Based on citation from the literature [5] it is estimate that contribution to the uncertainty equals:

$$u(\delta L_{\Omega}) = 0{,}5 \cdot 10^{-4} \cdot d \ , d \text{ in nm} \qquad (10)$$

3.4 Combined standard uncertainty $u_c(d_e)$

For previously described components the expression of combined standard uncertainty $u_c(d_e)$ is:

$$u_c(d_e) = (\sqrt{(9{,}6^2 + 0{,}233^2 d^2}) \ nm \ , d \text{ in nm} \qquad (11)$$

4. MONTE CARLO SIMULATIONS

Calculation of measurement uncertainty in the procedure of groove depth measurement by use of interferometric method has been performed applying Monte Carlo simulations (MCS).

MCS method is usable for calculation of measurement uncertainty as well as for validation of the results obtained by GUM procedure, and especially for determination of the interval for specific (given) probability *P*.

In this paper the estimation of measurement uncertainty by MCS method was performed by use of MathCad program. Probability density function of the output quantity d_e has been simulated by use of MCS method, according to expression:

$$d_e = (\kappa + F_r)\frac{\lambda}{2n_1} + \theta_e \cdot \alpha_e \cdot d + \delta L_{\Omega} \qquad (12)$$

Probability density functions g(*d*) of the output quantities *d* are presented in Fig. 1 and Fig. 2 for the groove of nominal depth 100 nm and 4000 nm.

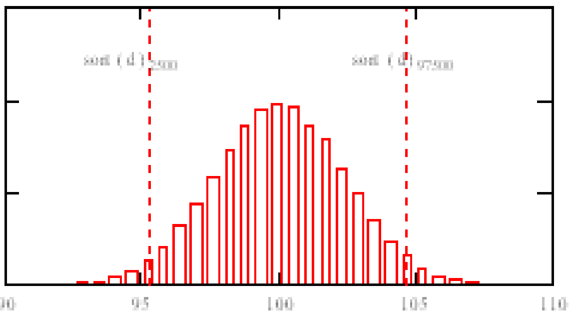

Fig. 2. Probability density function g(*d*) for 100 nm groove depth

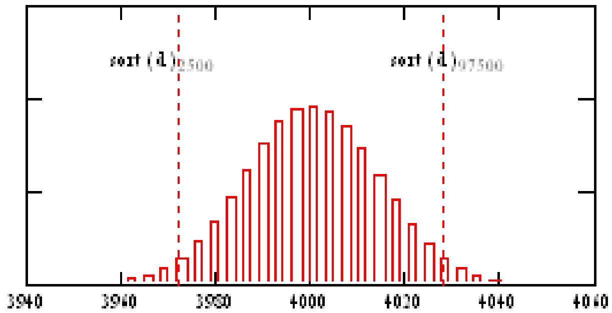

Fig. 3. Probability density function g(*d*) for 4000 nm groove depth

Probability density functions of the output quantities were obtained by convolution of input quantities distribution and use of $M = 100000$ simulations.

The estimated standard deviation of the output quantity *d* for the groove of nominal depth 100 nm amounts 2,4 nm as for the groove of nominal depth 4000 nm estimated standard deviation equals 14,3 nm, which confirms the uncertainties determined by the GUM method. The probability density functions g(*d*) of the output quantities *d* presented on Fig. 1 and Fig. 2 confirms the normal distributions of the output quantities.

The output quantity *d* for the groove of nominal depth 100 nm is within the interval:

$$(Y_{0,025} = 95{,}2 \text{ nm}; \ Y_{0,975} = 104{,}7 \text{ nm}) \text{ with } P = 95\%. \qquad (13)$$

The output quantity *d* for the groove of nominal depth 4000 nm is within the interval:

$$(Y_{0,025} = 3971{,}8 \text{ nm}; \ Y_{0,975} = 4028{,}0 \text{ nm}) \text{ with } P = 95\%. \qquad (14)$$

Expanded measurement uncertainty equals:

$$U = 4{,}4 \text{ nm} + 0{,}006 \cdot d \text{ nm}; \ d \text{ in nm}; \ k = 2; \ P = 95\% \qquad (15)$$

5. RESULTS AND DISCUSSION

The main problem of groove depth measurements by use of interferometric microscope, where the source of light is halogen lamp, is insufficient knowledge of the light wavelength. The data about the wavelength is crucial for calculation of the groove depth.

In Epival-interphako interferometric microscope solution of this problem was accomplished in two steps. As a first step, groove depth was measured using laser as a source of light. Based on measurement results of the real fractions and known value of the used laser wavelength, the depth of groove calculation was acquired.

The groove depth of the same standard was then measured, in the step number two, by use of halogen lamp as a source of light. The wavelength was indirectly defined through measurement results of the real fractions and previously calculated value of groove depth.

The results of groove depth measurements of three standards with different nominal groove depths obtained by use of interferometric microscope with two different light sources (halogen lamp and laser) are presented in Table 1.

For both used light sources, mean value of the groove depth d and estimated standard deviation s has been determined for each standard. For groove depth measurement results where laser has been used for a light source expanded uncertainties (k=2) were calculated.

Table 1. Measurement results for two different light sources

Nominal, nm	laser			halogen lamp	
	Value, nm	s, nm	U, nm	Value, nm	s, nm
100	108	1	5	111	2
800	865	4	10	867	7
2700	2675	10	21	2706	21

For a number of different roughness measurement devices where measurement traceability is not assured through laser, previously described procedure could be the way of traceability assurance up to the very top i.e. definition of meter.

6. CONCLUSIONS

Epival-Interphako interferometric microscope was modified by CCD camera installation and replacement of used light source from halogen lamp to laser.

This paper presents summary of influencing quantities in the procedure of groove depth d measurement by use of modified interferometric microscope. The expression of mathematical model describing the dependence of measured depth of groove d in relationship with all of the influencing components has been presented.

Analysis of influencing quantities has been done based on guidelines for calculation of measurement uncertainty: ISO Guide to the Expression of Uncertainty in Measurement (GUM) and Supplement 1 to the GUM – Propagation of distribution using a Monte Carlo method.

The main problem of groove depth measurements by use of roughness measurement devices, where the source of light is halogen lamp, is insufficient knowledge of the light wavelength. In this Paper, solution of that problem for Epival-interphako interferometric microscope has been presented. Described solution is applicable for a number of different roughness measurement devices where measurement traceability is not assured through laser. For those measurement devices described procedure could be the way of traceability assurance up to the very top i.e. definition of meter.

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